

## SOLUTION TO QUIZ 6

MATH 241

Solve the following integral equation for function  $f$ :

$$\int_0^{\infty} f(x) \sin \alpha x dx = \begin{cases} \alpha & 0 < \alpha < 1 \\ 0 & \alpha > 1 \end{cases}$$

*Proof.* By inverse Fourier sine transform,

$$\begin{aligned} f(x) &= \frac{2}{\pi} \int_0^1 \alpha \sin \alpha x d\alpha \\ &= \frac{2}{\pi} \int_0^1 \alpha d\left(-\frac{\cos \alpha x}{x}\right) \\ &= \frac{2}{\pi} \left[ -\left(\alpha \frac{\cos \alpha x}{x}\right) \Big|_{\alpha=0}^1 + \frac{1}{x} \int_0^1 \cos \alpha x d\alpha \right] \\ &= \frac{2}{\pi} \left[ -\frac{\cos x}{x} + \frac{1}{x^2} \sin \alpha x \Big|_{\alpha=0}^1 \right] \\ &= \frac{2}{\pi} \left[ -\frac{\cos x}{x} + \frac{\sin x}{x^2} \right] \end{aligned}$$

□